Exercise 70

After an antibiotic tablet is taken, the concentration of the antibiotic in the bloodstream is modeled by the function

$$C(t) = 8(e^{-0.4t} - e^{-0.6t})$$

where the time t is measured in hours and C is measured in μg/mL. What is the maximum concentration of the antibiotic during the first 12 hours?

Solution

The domain of the function is $0 \leq t \leq 12$. Take the derivative.

$$\begin{aligned} C'(t) &= \frac{d}{dt} 8(e^{-0.4t} - e^{-0.6t}) \\ &= 8\left[\frac{d}{dt}(e^{-0.4t}) - \frac{d}{dt}(e^{-0.6t})\right] \\ &= 8\left[(e^{-0.4t}) \cdot \frac{d}{dt}(-0.4t) - (e^{-0.6t}) \cdot \frac{d}{dt}(-0.6t)\right] \\ &= 8[(e^{-0.4t} \cdot (-0.4) - (e^{-0.6t}) \cdot (-0.6t)] \\ &= 8(-0.4e^{-0.4t} + 0.6e^{-0.6t}) \\ &= -3.2e^{-0.4t} + 4.8e^{-0.6t} \end{aligned}$$

Set $C'(t) = 0$ and solve for t.

$$-3.2e^{-0.4t} + 4.8e^{-0.6t} = 0$$

$$4.8e^{-0.6t} = 3.2e^{-0.4t}$$

$$\frac{4.8}{3.2} = e^{0.2t}$$

$$\ln \frac{4.8}{3.2} = \ln e^{0.2t}$$

$$\ln \frac{4.8}{3.2} = 0.2t \ln e$$

$$t = \frac{1}{0.2} \ln \frac{4.8}{3.2} \approx 2.02733 \text{ hours}$$

$t = (1/0.2)\ln(4.8/3.2)$ is within the interval $0 \leq t \leq 12$, so evaluate the function here.

$$C\left(\frac{1}{0.2} \ln \frac{4.8}{3.2}\right) = 8\left[e^{-0.4\left(\frac{1}{0.2} \ln \frac{4.8}{3.2}\right)} - e^{-0.6\left(\frac{1}{0.2} \ln \frac{4.8}{3.2}\right)}\right] \approx 1.18519 \, \frac{\mu\text{g}}{\text{mL}} \qquad \text{(absolute maximum)}$$

Evaluate the function at the endpoints.

$$C(0) = 8[e^{-0.4(0)} - e^{-0.6(0)}] = 0 \qquad \text{(absolute minimum)}$$

$$C(12) = 8[e^{-0.4(12)} - e^{-0.6(12)}] \approx 0.0598653$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $0 \le t \le 12$. The graph below illustrates these results.